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Semiclassical Expanding Discrete Space-Times[1]

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Given the close ties between general relativity and geometry one might reasonably expect that quantum effects associated with gravitation might also be tied to the geometry of space-time, namely, to some sort of discreteness in space-time itself. In particular we suppose space-time to consist of a discrete lattice of points rather than the usual continuum. Since astronomical evidence seems to suggest that the universe is expanding, we also demand that the lattice is expanding. Some of the implications of such a model are that the proton should presently be stable, and the universe should be closed although the mechanism for closure is quantum mechanical.

1. INTRODUCTION

The analogies between the general theory of relativity (GR) and non-Euclidean geometry are very strong (Misner et al., 1970). Indeed from a geometrical viewpoint, the gravitational field strength tensor itself is nothing more than a description of the shape of space-time. Reasoning geometrically then, one might expect to associate quantum gravity with quantized geometry ('t Hooft, 1979). As a first approximation we shall simply replace the continuum by a discrete lattice of points with spacing l which will depend on matter fields ψ. We note that the subject of discrete space-times has been discussed previously by many authors (Macrae, 1981; Myrheim, 1978; Townsend, 1977; Finkelstein, 1969, 1972a, 1972b, 1974; Bopp, 1967).

The existence of such a lattice structure in space-time could have pronounced effects upon physics. For example, particle momenta are constrained by the relationship: $p \leqslant h/l$, so any particles not satisfying this relationship are forbidden. Thus the lattice spacing imposes a natural

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"ultraviolet cutoff" on the physics taking place on the lattice. However, we envision that the matter fields will strongly influence the lattice and partially mollify this effect, but not entirely.

Since the current astronomical data strongly suggest that the universe is expanding, one might expect that the lattice spacing l, and hence the lattice itself, must also be expanding. If l *is* a function of time which has been steadily increasing since the Big Bang, then the "ultraviolet cutoff" has been steadily *decreasing* since the Big Bang as well! Such a change in cutoff will have important implications for grand unified field theories (Georgi and Glashow, 1974; Weinberg, 1980; Salam, 1980; Glashow, 1980) and indeed for the ultimate fate of the universe as well.

We discuss our model in Section 2. We present the implications of our model for proton decay in Section 3, the expansion and eventual collapse of the universe in Section 4, and astrophysics in Section 5. Our speculations are given in Section 6, and we summarize our predictions in Section 7.

2. THE CURRENT VALUE OF THE LATTICE SPACING

Shortly after the Big Bang many features associated with the present-day universe became "frozen" into the geometry. In particular, we assume that by the Planck time ($t^* \sim 10^{-43}$ s) the fundamental constants of nature (G, h, c, etc.) had obtained their present values. The question then arises: is there any natural choice for the lattice spacing at t^*? One obvious choice is the Planck length[3] ($l^* \sim 10^{-33}$ cm), so we take $l(t^*) \equiv l^*$ and assume l to have been increasing ever since. [We use geometrized units in which $G = c = 1$ for convenience throughout the remainder of the article.]

Here we make the crucial assumption that the Einstein equations predict the large-scale behavior of the universe down to, but not below, the graniness of space-time. Thus we would not attempt to push the Einstein theory below the present-epoch value of l (though in the early universe it could perhaps be pushed to near l^*). This means that we can use the Einstein equations to predict the behavior of $l > l^*$ (neglecting matter fields ψ). Alternately, we say that for the general features of space-time, then l represents the small-scale limits of applicability of the Einstein equations.

For simplicity we represent the universe by a Robertson–Walker metric with $k = 0$, and we also take the cosmological constant $\Lambda = 0$. From Einstein's equations one finds for stiff matter that the scale factor $R(t)$

[3]Many authors have assumed l^* is the invariable lattice spacing, e.g., the recent papers by Macrae (1981a,b,c).

satisfies the relation

$$R(t)=\left\{1+(t-t^*)[6\pi G\rho(t^*)]^{1/2}\right\}^{1/3}R(t^*) \tag{1}$$

where $\rho(t^*)$ is the density at time t^*.

For a universe undergoing a scale expansion, a distance d at two different times t and t' is related by

$$d(t)=\frac{R(t)}{R(t')}d(t') \tag{2}$$

where $R(t)$ and $R(t')$ are the scale factors at the times t and t', respectively. Thus we expect the lattice spacing $l(t)$ to be given by

$$l(t)=\left\{1+(t-t^*)\left[6\pi G\rho(t^*)\right]^{1/2}\right\}^{1/3}l^* \tag{3}$$

In obtaining equation (3), we necessarily suppressed the interactions of the fields ψ. We can approximate the effects of the fields ψ by assuming that the equation of stiff matter, assumed for the initial stages of the universe, continues to apply to the lattice. This means that we must now compare the initial density to the density of present day matter which we take to be the matter interior to a proton. The equation for the density of stiff matter scales as

$$\frac{\rho}{\rho^*}=\left(\frac{l}{l^*}\right)^{-6} \tag{4}$$

where ρ^* is the Planckian density, $2.27\times10^{64}/\mathrm{cm}^2$. The proton's electromagnetic radius $r_p\sim0.8\times10^{-13}$ cm, so that its mass density is about $5.78\times10^{-14}/\mathrm{cm}^2$. We obtain

$$l(\psi)=1.37\times10^{-20}\ \mathrm{cm} \tag{5}$$

for the present day value of the lattice spacing in the *presence of strongly interacting matter fields* ψ.

If we assumed that the Hubble constant scales with the lattice, then we obtain

$$H=\frac{c}{l^*}\left(\frac{l}{l^*}\right)^{-3} \tag{6}$$

for consistency. This predicts that the average lattice spacing for the universe with a *uniform* mass density would be $l_{\mathrm{univ}}\sim2.9\times10^{-13}$ cm. Thus we would not attempt to use GR then for distances $\leqslant10^{-13}$ cm.

3. PROTON DECAY

If our model is to have any relevance, we take the lattice spacing $l(\psi)$ for stiff matter, given by equation (5), to be the present day minimum size of the lattice, i.e., the limiting range for interactions. Alternately this corresponds to an ultraviolet cutoff of about 9000 TeV.[4]

Current grand unified theories of the weak, electromagnetic, and strong interactions predict proton decay through the agency of supermassive particles called leptoquarks. The force which these particles mediate has a range of about 10^{-29} cm, whereas $l(T, \psi) \sim 10^{-20}$ cm, where T is the present epoch. Thus since the current lattice spacing is about 10^9 times greater than the range of the force responsible for the decay, we would *not* expect proton decay to be observed. Note though, that for times shortly after the Big Bang the lattice spacing is still less than the range of the leptoquark force, so proton decay would still be allowed then.

Since l is still much smaller than the "unification length" (10^{-16} cm in most models) (Georgi and Glashow, 1980) for the electroweak theory, our model would *not* interfere with *that* theory's predictions in any extreme way.

4. THE EXPANSION AND CONTRACTION OF THE UNIVERSE

Suppose that we have an expanding universe and $\rho < \rho_{\text{crit}}$, where ρ_{crit} is the density needed to close the universe. Under these circumstances one normally supposes that the expansion will continue indefinitely. Consider, however, that at some point $l(t, \psi)$ will exceed 10^{-13} cm within matter. Since the range of the nuclear forces is also about 10^{-13} cm, once the lattice spacing reaches this limit the nuclear forces should "turn off." We will investigate what this means below. It is interesting to note that l_{univ} is also of the order of 10^{-13} cm. We can only speculate and conjecture that the role of geometry for the strong interactions is more important than we previously suspected. At this stage there are several scenarios depending on the nature of quarks and their interaction. Unfortunately the exact nature of the quark is not known. Depending on the model the mass of quarks are from a few MeV to hundreds of GeV, and whether there can be free quarks is still an open question. Let us assume here the values for the quark masses in the neighborhood of a few hundred MeV given by the grand unified theories. (The value of the quark mass is not critical to the arguments that follow.) As

[4] The ultraviolet cutoff refers to the interaction and exchange of particles in the center-of-mass (momentum) system and should not be confused with kinematical effects such as high-energy cosmic rays in the laboratory frame.

the lattice expands, the average separation between quarks also expands (c.f. also Section 6). When the quarks are sufficiently far apart, the fields will eventually become strong enough to generate spontaneous particle production. Further lattice expansion should lead to copious particle production. The energy for particle production, which is provided by the expanding lattice, slows or stops the expansion. And this effect, together with the greater matter–energy density starts the universe collapsing again.

Soon after the lattice begins contracting l becomes less than 10^{-13} cm and particle production ceases. Contraction presumably continues until conditions again approach those of the Big Bang.

5. ASTROPHYSICAL IMPLICATIONS

Quasars, the most distant objects which we observe, seem to be as much as 5×10^{27} cm away from us (Weinberg, 1972). Consequently the radiation which we observe from them must have been emitted about 2×10^{17} s ago. If we denote by t_q the time at which a typical quasar emits a signal, then since $t_q \sim \frac{1}{3}T$ we deduce by equation (3) that $l(T)\sim 1.5l(t_q)$ (we are neglecting matter effects, which would also contribute). Since the red shifts of the most distant quasars are of the same order, we suggest that the expanding lattice be interpreted as the source of the cosmological red shift. We call this the lattice Doppler shift.

6. DISCUSSION

We suggest, along with others, that the discrete lattice model represents a prototype model for quantizing space-time. We have indicated how a reasonable model would involve gravitational effects of matter on the lattice itself, i.e., l must be a function of the matter distribution as well as the matter fields. The immediate implication is that a lattice spacing dependent on matter would allow for additional red shifts in the case of compact objects such as quasars. The model also provides a possible explanation for the difference in red shift for different cosmological objects within the same cluster of galaxies.

It is well known that invoking a lattice model allows one to calculate many of the desirable features of the quantum chromodynamical theory of strongly interacting particles (Kogut and Suskind, 1975; Wilson, 1974). We suggest, as others ('t Hooft, 1979) have done, that the *reality* of the lattice be taken more seriously. We speculate that the quarks in the neighborhood of a proton (for example) occupy lattice points. The number of lattice sites is approximately given by $N\sim(r_p/l)^3\sim10^{20}$. The quarks are assumed free to occupy any of them. Another way of looking at this is to consider the following: outside the range of the strong fields inside the proton, $N \sim (r_{p/l_{\text{univ}}}) \sim 1$. Thus the overwhelming probability is that the quark remains inside the proton. If the quarks move too far apart the lattice will tend to expand toward the ambient value l_{univ}. To do this the lattice must absorb energy from its surroundings (collisions, decays, virtual particle exchanges, etc.). As a result quark production is energetically suppressed, but quark–antiquark pair production is favored, since it tends to maintain the density of states (lattice sites) within the range of the strong force.

7. PREDICTIONS

We have discussed a semiclassical model for space-time in which the usual continuum is replaced by a lattice. Since experimental evidence suggests the universe to be expanding we have demanded that the lattice expand also. We conclude by summarizing the predictions of our expanding lattice model:

- protons are presently stable
- gravitational red shifts are proportional to change in lattice spacing between emitter and absorber
- the universe is closed.

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REFERENCES

Bopp, F. (1967). *Zeitschrift für Physik*, **200**, 117, 133.
Finkelstein, D. (1969). *Physical Review*, **184**, 1261.
Finkelstein, D. (1972). *Physical Review D*, **5**, 320, 2922.
Finkelstein, D. (1974). *Physical Review D*, **9**, 2219.
Georgi, H., and Glashow, S. L. (1974). *Physical Review Letters*, **32**, 438.
Georgi, H., and Glashow, S. L. (1980). *Physics Today*, **33**, 30.
Glashow, S. L. (1980). *Reviews of Modern Physics*, **52**, 539.
Hooft, G. 't (1979). in *Recent Developments in Gravitation*, Cargèse, 1978, Plenum Press, New York, p. 323.
Kogut, J., and Susskind, L. (1975). *Physical Review D*, **11**, 395.
Macrae, K. I. (1981). *Physical Review D*, **23**, 886, 893, 900.

Misner, C. W., Thorne, K. S., and Wheeler, J. A. (1970). *Gravitation*, Freeman, San Francisco.
Myrheim, J. (1978). Ref. T4 2538-CERN.
Salam, A. (1980). *Reviews of Modern Physics*, **52**, 525.
Townsend, P. K. (1977). *Physical Review D*, **15**, 2795.
Weinberg, S. (1972). *Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity*, Wiley, New York.
Weinberg, S. (1980). *Reviews of Modern Physics*, **52**, 515.
Wilson, K. G. (1974). *Physical Review D*, **10**, 2445.